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Short communication

Estimation of the state-of-charge of lead–acid batteries used in electric scooters

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Abstract

The use of electric scooters is being advocated by the government of Taiwan. An accurate indication of the state-of-charge (SoC) of the battery (lead–acid) is essential to let the driver of the electric scooter know exactly the residual amount what amount of usable capacity. This will result in greater confidence in using the electric scooter, and prevents the driver from becoming stranded with an inoperative machine. The basic idea of this study is to derive a mathematical model that describes the battery voltage as a function of discharge current and depth-of-discharge (DOD). The parameters of this model are identified in real time. Then, the SoC is estimated by means of the Newton–Raphson (N–R) method. The proposed algorithm enables the estimated SoC to be adjusted for changes in the behaviour of the battery. The effectiveness of the proposed method is verified using field-test data obtained from driving an electric scooter on various routes. The results show that the adopted method improves the accuracy in estimating the battery of SoC.

Keywords: State-of-charge; Electric scooter; Lead–acid battery; Newton–Raphson method; Recursive least-squares method; Mathematical model

1. Introduction

Just as it is important to know the amount of fuel remaining in an automobile, the state-of-charge (SoC) of the battery is essential information in electric scooter operation. If the SoC can be estimated accurately, the user will know exactly the amount of power that the battery is capable of providing. This naturally results in a greater confidence in the power source, and will allow proper handling of the electric scooter.

During discharge, the energy capability of a battery depends on a number of parameters that include discharge current, temperature, battery age, cut-off voltage, and service history (previous charge and discharge) [1]. The SoC of a battery is a complex non-linear function of these parameters. In practice, direct measurement of some of these parameters is either impossible or financially prohibitive.

Traditionally, there are several practical methods available to monitor lead–acid batteries and translate the resulting information into a prediction of performance capability. The specific gravity method, the open-circuit voltage method, and the coulometric measurement method are some examples. Specific gravity is a direct indication of the SoC, because it shows the concentration of acid in the electrolyte (note, acid is consumed during the discharge of a lead–acid battery). It is impossible, however, to measure specific gravity continuously under driving conditions. Furthermore, the use of gelled electrolytes and hermetically sealed batteries preclude the application of density measurements for evaluating the SoC. The open-circuit voltage of a lead–acid battery is a function of the concentration of acid at the plates. The fully stabilized open-circuit voltage is an accurate indicator of the SoC with little dependency on the temperature or the past history of a battery, but to reach a stable open-circuit voltage the battery must be at rest for several hours with no load. Therefore, it is not a realistic option for an electric scooter. The coulometric method measures the amount of ampere-hours taken out of a battery to determine the SoC. Correction factors are required for different discharge rates and ambient temperatures [2]. This technique does provide a relatively

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accurate short-term indication of the SoC, but accumulation of error over a longer period makes it impractical to be used by itself.

Although all of these methods have provided some degree of success in areas of limited scope, batteries are complex and dynamic electrochemical systems with performances that are dependent on a combination of all of these parameters. Measurement of a single performance variable provides an incomplete picture of the state of the battery, and therefore, a limited degree of accuracy.

Recently, a model-based technique has been proposed as an alternative for determining battery SoC. Various models have been developed based on the characteristic of discharge voltage versus time (or discharge voltage versus SoC). Curve-fitting models are used to match the discharge voltage versus time or discharge voltage versus SoC curve with either a polynomial [3], an exponential [4] or a parabolic [5] curve. Some models employ more complicated hybrid approaches [6,7]. The major disadvantage of these models is that a parameter set is required for each combination of discharge conditions and battery type. These parametric sets are obtained through an exhaustive series of discharge tests. Also, a drastic change in battery behaviour may take place due to ageing, thermal stress, or some other operating reasons. For more accurate SoC estimation, changes in battery behaviour should be taken into account whenever a discharge occurs. These approaches suffer from this requirement.

The present study attempts to develop an adaptive SoC estimating method. A discharge voltage versus depth-of-discharge (DoD) and discharging current representation is derived to represent the battery discharge characteristics. The parametric set is obtained in real time through an adaptive parameter identification technique, which enables the estimation to be adjusted to different batteries and also accommodates ageing effects. Road tests data obtained from driving an electric scooter on various routes are used to evaluate the effectiveness of the proposed method. In this investigation, the SoC refers to the available capacity (in ampere-hours) remaining in the battery.

2. Design of adaptive SoC estimator

A mathematical model is derived to describe the battery voltage as a function of discharge current and DoD. During battery operation, the model continuously updates its coefficients by applying an adaptive parameter identification technique. The SoC is then estimated by employing the Newton–Raphson (NR) method to predict the final DoD at which the cut-off voltage is reached. The principle of the proposed method is illustrated in Fig. 1.

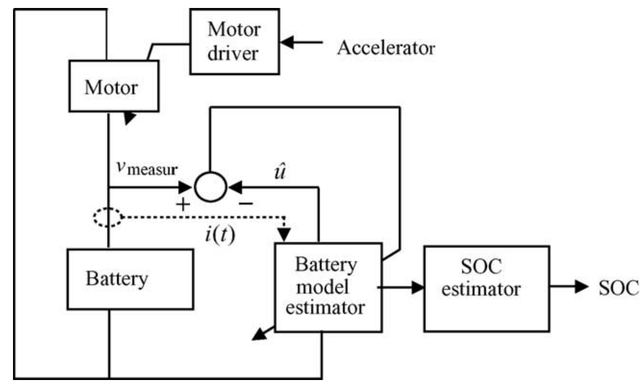

Fig. 1. Principle of the adaptive battery state-of-charge estimation.

2.1. Battery model identification

Traditionally, a non-linear function in the time domain that describes the relationship between battery voltage and discharge conditions has been employed to estimate battery SoC. This representation is not robust, because it is sensitive to changes in the discharge conditions. A more robust model can be obtained by representing the discharge in the SoC domain instead of the discharge time [8]. Because the SoC is unknown during battery discharge, these models should be determined via an exhaustive series of discharge tests. This enables the model to be adaptive to different batteries and ageing effects are avoided. The following model represents the discharge in an equivalent DoD domain:

$$\begin{aligned}\hat{u}(i(t), s) &= f(i(t), s) = \hat{c}_1 + \hat{c}_2 f_1(s) + \hat{c}_3 i(t) + \hat{c}_4 i(t) f_2(s), \\ &= x^{\mathrm{T}}\hat{\theta},\end{aligned} \tag{1}$$

where $x = [1, f_1, (s), i, i \cdot f_2(s)]^{\mathrm{T}}$ represents the system's model and $\hat{\theta} = [\hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4]^{\mathrm{T}}$ is the parametric vector to be estimated. The variable $\hat{u}(i, s)$ in Eq. (1) is the estimated battery voltage, $i(t)$ the measured discharging current of the battery, and s is the DoD which is obtainable through the discharge cycle and is calculated by dividing the coulometric measurement by the total charge ($\mathrm{AH}_{\mathrm{charged}}$) supplied to the battery during the recharge cycle, i.e.,

$$s = \frac{\int_0^t i \, \mathrm{d}\tau}{\mathrm{AH}_{\mathrm{charged}}} \tag{2}$$

Because of the strong dependence of the battery voltage on the DoD during discharge, Eq. (1) can be shown to be successful in tracking the battery behaviour as will be shown later. In Eq. (1), the functions $f_1(s)$ and $f_2(s)$ are exponential-type curves determined by fitting them to average battery discharge behaviour. When the discharge parameters are measured, the coefficients c_1–c_4 in $\hat{\theta}$ are identified in real time using the technique described briefly as follows.

At the mth measured point, Eq. (1) can be expressed as:

$$\hat{u}_m = x_m^{\mathrm{T}} \hat{\theta}_m \tag{3}$$

where x_m and $\hat{\theta}_m$ are the mth measurements of the vectors x and $\hat{\theta}$, respectively. If the mth measured data are used, the parameters in Eq. (1) can be estimated by the least-squares method, i.e.,

$$\hat{\theta}_m = (x_m^{\mathrm{T}} x_m)^{-1} x_m^{\mathrm{T}} v_m \tag{4}$$

When a new measurement, the $(m+1)$th data, is obtained, it is apparent that to obtain $\hat{\theta}_{m+1}$, we must invert an 4×4 matrix. To preclude the matrix inversion and to improve our parameter estimates by making use of this new information, a recursive algorithm [9,10] is applied when experimental data are being gathered continuously.

With a new measurement, the predicted battery voltage $(x_{m+1}^{\mathrm{T}} \hat{\theta}_m)$ using the previous estimation is compared with the actual value (u_{m+1}). The resulting error is expressed as:

$$e_{m+1} = u_{m+1} - x_{m+1}^{\mathrm{T}} \hat{\theta}_m. \tag{5}$$

Then, the new parameter estimation is calculated as:

$$\hat{\theta}_{m+1} = \hat{\theta}_m + \gamma_{m+1} P_m x_{m+1} e_{m+1} \tag{6}$$

with

$$P_{m+1} = \frac{1}{\lambda}[P_m - \gamma_{m+1} P_m x_{m+1} x_{m+1}^{\mathrm{T}} P_m], \tag{7}$$

and

$$\gamma_{m+1} = \frac{1}{1 + x_{m+1}^{\mathrm{T}} P_m x_{m+1}} \tag{8}$$

where the matrix x is the identified model and $\hat{\theta}$ is the coefficient matrix of x to be calculated. By starting with an initial estimate $\hat{\theta}(0)$ and the corresponding $P(0)$, $\hat{\theta}$ can be sequentially updated while new data are being continuously obtained. With this recursive algorithm, the estimates can be updated step-by-step without repeatedly computing the matrix solution of Eq. (4). This on-line algorithm allows the estimate $\hat{\theta}$ to be updated easily as the number of measurements increases. Thus, it is possible to track system parameters that vary slowly.

2.2. SoC estimate based on battery model

A block diagram of the steps for performing the SoC estimation is given in Fig. 2. The input data include the measured battery voltage (u), the measured amperes flowing out of the battery (i), the total ampere-hours charged to the batteries ($\mathrm{AH}_{\mathrm{charged}}$), the accumulated discharging time (t), and the cut-off voltage u_{f} specified by the battery supplier. Step P1 computes the average power delivered by the battery over the time period from the beginning of the discharge to the present time. Step P2 determines the DoD (denoted by s) at the present time t. Step P3 calculates the final discharging current (i_{f}) when the voltage of battery under the present consuming power (PW) drops to the cut-off voltage (u_{f}). Step P4 performs the battery model identification using the adaptive parameter identification technique described above. The identified battery model and the calculated i_{f} are fed to step P5 to estimate how deep the battery will discharge, i.e., the value of s_{f}, under the present consuming power. This is accomplished by using the Newton–Raphson method as follows.

For a battery with its behaviour described by the mathematical model, $\hat{u}(i, s) = f(i, s)$, and with a discharge rate of i_{f}, the value of s_{f} at which the battery voltage falls to the cut-off voltage (u_{f}) can be obtained by finding the unique positive solution of

$$g(i_{\mathrm{f}}, s) = f(i_{\mathrm{f}}, s) - u_{\mathrm{f}} = 0 \tag{9}$$

According to the NR method, the root s_{f} of $g(i_{\mathrm{f}}, s)$ can be obtained by using the iteration:

$$s_{k+1} = s_k - \frac{g(i_{\mathrm{f}}, s_k)}{g'(i_{\mathrm{f}}, s_k)}. \tag{10}$$

In this study, since s_{f} of the present cycle is used as the initial guess value for the next cycle, the prediction can always be completed within three iterations. At the final stage of Fig. 2,

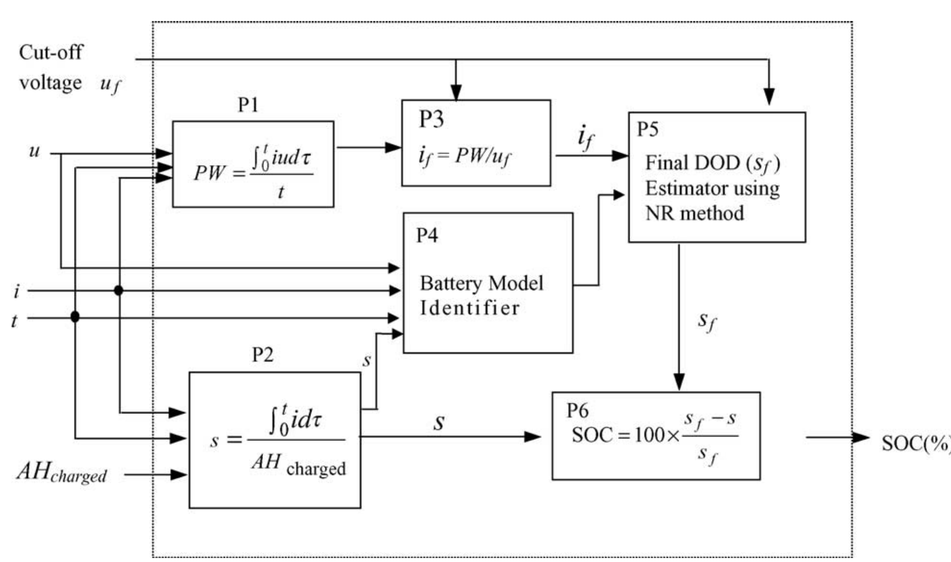

Fig. 2. Signal flow in SoC estimation.

the estimated value of s_f and the present value of s are fed to step P6 to calculate the SoC (%):

$$\mathrm{SoC} = 100 \times \frac{s_f - s}{s_f}. \tag{11}$$

The estimate of the SoC is calculated by iteration during the $[t - T, t]$ period. The SoC estimated by Eq. (11) is referred to as the 'residual capacity'. As an example, suppose that a battery charged with 30 Ah is discharged under dynamic loads. At the beginning of discharge, the SoC and s are set to be 100% and 0, respectively. As time proceeds, the SoC is decreased with increase in s. At the time instance t with certain amount of consuming power, the battery is discharged to the depth of $s = 0.2$. Supposed that the value of s_f is predicted to be 0.8, then the SoC is obtained as $100 \times (0.8 - 0.2)/0.8 = 75\%$. $s_f = 0.8$ means that the present capacity of the battery has been lowered from 30 to 24 Ah. The usable capacity is then $30 \times 0.8 = 24$ Ah at the present time. Thus, the remaining available capacity is $24 \times 0.75 = 18$ Ah.

3. Field data test results

Simulation tests were made for confirmation of the effectiveness of the proposed algorithm using the battery voltage and current data measured from city driving including many starts and stops. The "Emuda" electric scooter by Shang-wei Co. Ltd., Taiwan, was used for tests because of its availability. It was powered by a 48 V lead–acid battery set (four 12 V battery units connected in series) and was equipped with 0.58 kW dc brushless motor that drove the rear wheel through a continuously variable transmission. CSB EVX-12400 deep-discharge Valve-regulated lead–acid batteries were used in the tests and were each rated at 40 Ah. The batteries were fairly new at the time of testing. The batteries were fully charged before each discharge cycle by means of a 48 V charger that was built into the electric scooter. The charging schedule was a combination of two constant-power/constant-voltage steps, and the charging time was about 6–8 h.

After the batteries had been charged completely, discharge cycles were begun by driving the electric scooter in various traffic conditions and were terminated when any one of the four battery units reached the cut-off voltage of 9.6 V. In all the road tests, the driving mode always consisted of accelerating, decelerating and intermediate stopping periods according to the traffic. Thus, the batteries are not discharged at a constant rate, but at a time-varying rate in the range of 0–100 A as determined by the slope of the road, driving speeds, and traffic. Due to the load variations, the fluctuations in battery current and battery voltage provide an excellent opportunity to capture the dynamic characteristics of the batteries. As a result, the persistent excitation requirement is satisfied for the parameter identification process.

During the charge–discharge cycles, parameters such as the battery current, voltage, and the speed of the scooter are measured and recorded every 0.5 s via a hand-held data recorder. All the data records are stored in a personal computer and numbered according to the sequence of the consecutive cycles.

Using the field-test data, SoC estimations were carried out by feeding the data into the proposed algorithm. During the calculations, the parameter identification and SoC estimation are conducted in real time as the data are being recorded. This is to simulate the driving schedule of the scooter. For the first discharge cycle, the initial values for parameter identification are selected arbitrarily. Thus, the performance of the resulting SoC estimate is unsatisfactory. As time proceeds, however, the accuracy will be improved and stabilized because the identified results of the previous cycles are used as the initial values for the following cycle identifications.

The results of the estimates for one of the four series-connected batteries using the data of the third discharge cycle are shown in Fig. 3. The estimated voltage tracks the measured value successfully, and the estimated SoC shows a percentage error of less than 7% throughout the discharge. In this study, the SoC estimations were made for the four batteries individually, since different batteries will exhibit different characteristics. For example, the capacity of one battery may be exhausted while the others still retain some capacity. Due to the average effect of the battery string, this difference may be smoothed out.

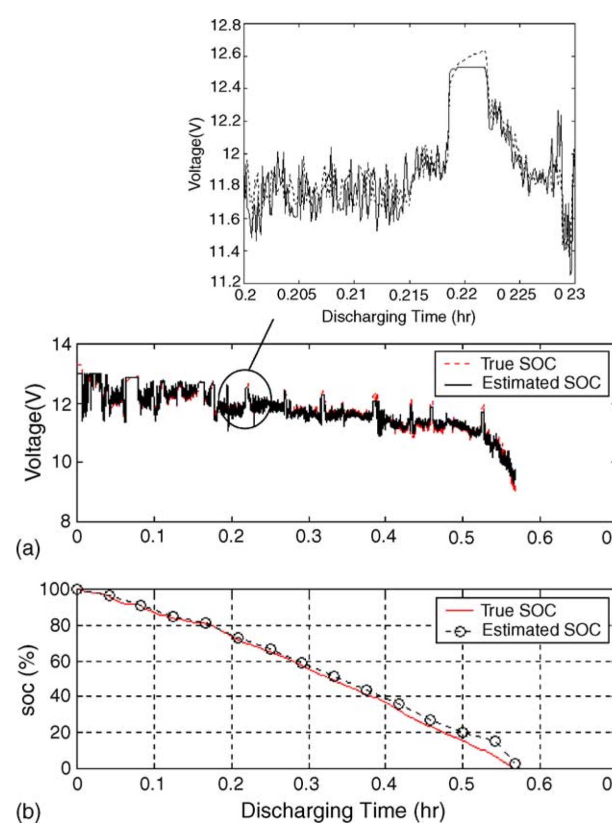

Fig. 3. (a) Comparison between measured battery voltage and estimated voltage signals. (b) Comparison between actual and estimated SoC.

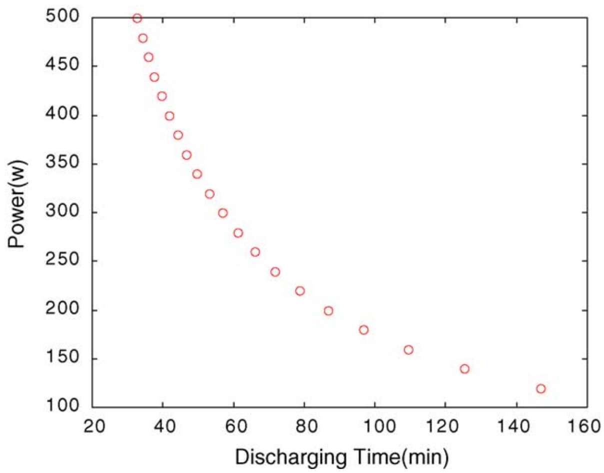

Fig. 4. Simulated constant-power discharge behaviour of CSB lead–acid battery using model given by Eq. (1).

Table 1
Constant-power discharge characteristics of CSB EVX-12400 battery at 25 °C

Discharge time	Discharging power (W)
30 min	457
60 min	275
90 min	201
2 h	170
3 h	117
4 h	93.9

All the values are average values.

The predicted discharge times for various discharge powers using the battery model given by Eq. (1) are presented in Fig. 4. The resulting predictions are in good agreement with the experimental data listed in Table 1 (provided by CSB) in which the discharges were conducted at 25 °C from a fully-charged state to an end voltage of 9.6 V.

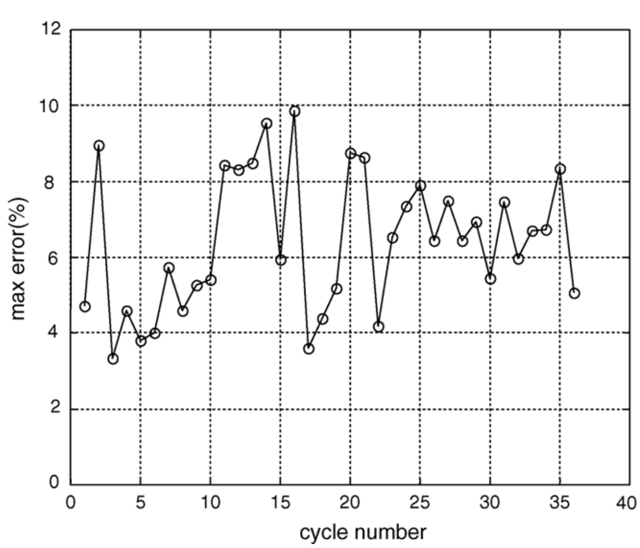

Fig. 5. Error distribution of SoC estimations for 36 data records obtained from the road tests over various routes.

To validate the effectiveness of the proposed method, SoC estimations were conducted for 36 data records obtained from road tests performed under various driving conditions. The error distribution of the SoC estimations for these data records is given in Fig. 5. The results demonstrate good performance of the estimations with a maximum error within 10%.

4. Conclusions

A new approach for estimating the SoC of a lead–acid battery during discharge is discussed. The accuracy is better than 90%. The approach utilizes a mathematical model that describes battery voltage as a function of discharge current and DoD. The coefficients of the model are identified in real time using measurable data during battery discharge. The SoC is then estimated through the prediction of the final DoD by using the battery model.

The algorithm for SoC estimation possesses a learning capability to accommodate ageing and behavioural changes of the battery. Simulation tests using battery voltage and current signals measured in road tests have been performed. The effectiveness and accuracy of the proposed method are confirmed. The proposed SoC estimator can be easily implemented with low-cost microprocessors.

In this approach, the SoC estimation is designed with emphasis on the discharge cycle only; the charge cycle is not considered. Further work will involve combining charge with discharge in SoC estimations to form a complete solution for battery monitoring and management. An attempt will also be made to predict the lifetime of a battery according to its history of deterioration.

Acknowledgement

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